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SPECULATIVE DYNAMICS AND FEEDBACK TRADING A NONLINEAR MODEL

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Abstract. This paper studies a discrete time market model under heterogeneous trading. In the market there are a market maker and three groups of traders: feedback traders, fundamental traders and noise traders. We propose a nonlinear cumulative demand process driving the asset price. Through simulations we find that the asset price moves in the direction of the fundamental value only if there are enough fundamental traders. Positive feedback strategies tend to increase the volatility of the price and move away the price from his fundamental value. In this case speculative bubbles and positive serial correlation in returns are observed.

1 Introduction

The standard efficient market model for asset prices based on a representative agent with a perfect information provides that all movements in asset prices can be accounted for news on fundamental values. On the other hand, the well known log-normal model is the product of the classical rational expectation assumption, see Kreps [11]. It follows that excess returns in asset price time-series should be no predictable and no sign of autocorrelation should be observed. However, one of the most discussed anomalies in the financial literature is the predictability of excess returns (see for instance Cutler Poterba and Summers [4], [5] and references therein).

In recent years much effort has been devoted to explain these anomalies invoking the presence of some elements of irrationality in the market. In this direction a number of market models under heterogeneous trading and learning has been proposed. According to certain authors the key assumption is the bounded rationality. Traders forecast the future prices by updating their expectations through a first order autoregressive learning mechanism, the so called adaptive expectations (see for instance Barucci and Landi [2], Cutler, Poterba and Summers [5]). In this framework predictability of excess returns and high volatility has been obtained. Furthermore, through some approximation arguments has been showed that the price process is driven by a mean reverting Ornstein-Ulenbeck process around the level given by the expected price process (see Föllmer and Schweizer [7], Giuli and Monte [8]).

The linearity assumption of the market demand function is a common feature of the above studies. Several authors have attempted to stress the linearity condition of demand proposing a number of nonlinear market models, see for instance Kaizoji [10] and references therein. In the present paper we aim to contribute to this debate studying a new discrete time model. First we assume that the trading mechanism is controlled by a market maker adjusting the asset prices according to the excess of demand function. This is a more sound behavior with respect to the classical market clearing condition. Then we introduce three classes of agents in the market, positive feedback traders, fundamental traders and liquidity traders. Feedback traders look over the past evolution of asset prices. They mechanically respond to price changes, buy when the price raises and sell when the price declines. Fundamental traders belief that the asset price will be attracted by a fundamental value. They buy or sell the risky asset according to whether this one is underestimated or overestimated with respect to the fundamental value. Finally, liquidity traders are pure noise in the market demand, buying or selling the risky asset only for their liquidity needs. Through simulations we show that predictability of excess returns and excess volatility can be generated in a nonlinear setting without learning.

The paper is organized as follows. In Section 2 we describe the financial market model. In Section 3 we study the model without noise and with only one group of traders. In Section 4 we simulate the global dynamics.

2 The Model

We consider a market where a risky asset with a well defined fundamental value is traded. We have in our mind a future, whose fundamental value is its terminal value. In the market there are three groups of agents: feedback traders, fundamental traders and noise traders. Feedback traders base their strategy on the past evolution of the asset price. We consider only positive feedback traders who buy if the price has increased and sell if the price has decreased. Fundamental traders forecast the future asset price on the basis of the distance between the asset price and his fundamental value. They buy or sell the risky asset according to whether their forecasts of asset future returns are higher or lower than a required return. Finally, noise traders are pure noise in the market demand, buying or selling the risky asset only for their liquidity needs. Following Kaizoji [10] we omit dividends and interest rates. We define the cumulative rate of the excess demand for the asset at discrete time k as

$$x_k = n\theta^{-1}(ax_k^{(1)} + bx_k^{(2)} + cx_k^{(3)})$$

where $x_k^{(1)}$, $x_k^{(2)}$, $x_k^{(3)}$ and a, b, c are the excess demand functions at time k and the proportions of the different groups of traders in the market, feedback, fundamental and noise respectively, n is the total number of traders and θ is the total number of listed shares.

In our model a market maker adjusts the asset price process in each time interval $(k, k+1)$. The market maker will raise or reduce the asset price according to whether the excess demand at time k is positive or negative. In particular we assume that

$$p_{k+1} - p_k = \delta x_k \tag{1}$$

where p_k is the logarithm of the risky asset price at the instant k and $\delta > 0$ is the speed of the adjustment of the price.

In this general setting we assume that the cumulative rate of the excess demand is changing in observance of the following rule

$$x_{k+1} = \sigma z_k + \omega\left((1 - x_k)f_{k+1}^{+} - (1 + x_k)f_{k+1}^{-}\right) \tag{2}$$

where (z_k) is a sequence of independent and normally distributed real random variables with zero mean and unit variance, the transition functions f^+ and f^- are defined as

$$f_{k+1}^{+} = \mu + \exp(\alpha(p_{k+1} - p_k) + \beta(\hat{p}_{k+1} - p_{k+1})) \tag{3}$$

and

$$f_{k+1}^{-} = \mu + \exp(-\alpha(p_{k+1} - p_k) - \beta(\hat{p}_{k+1} - p_{k+1})) \tag{4}$$

$\hat{p}_k$ is the logarithm of the forecasted risky asset price at the instant k, which is expected at the instant $k + 1$, the nonnegative coefficients α, β and σ model the relevance of the different groups of traders in terms of the proportions and of the elasticities of demands with respect to the price, μ and ω are positive parameters with $\omega < 1/3$.

We also assume that fundamental traders believe that the actual asset price will be attracted to a reference fundamental level e^{p^*}, see Föllmer and Schweizer [7]. Specifically, let us assume that fundamental traders forecast the risky asset price according to the simple mechanism

$$\hat{p}_k = p_k + \nu(p^* - p_k) \tag{5}$$

for a suitable updating coefficient $0 \le \nu \le 1$. Equation (5) states that for fundamental traders the price will move in the direction of the fair price e^{p^*}. From (1), (2) and (5) we deduce the following discrete evolution model

$$\begin{cases} p_{k+1} - p_k = \delta x_k \\ x_{k+1} = \sigma z_k + \omega\left((1 - x_k) f^+_{k+1} - (1 + x_k) f^-_{k+1}\right) \\ \hat{p}_{k+1} = p_{k+1} + \nu(p^* - p_{k+1}) \end{cases}$$

Substituting (5) and (1) in (3) and (4) we extrapolate the following dynamics that we will study

$$\begin{cases} p_{k+1} - p_k = \delta x_k \\ x_{k+1} = \sigma z_k + \omega\left[(1 - x_k)(\mu + \exp(\alpha\delta x_k + \beta\nu(p^* - p_{k+1})))\right. \\ \qquad\quad \left. - (1 + x_k)(\mu + \exp(-\alpha\delta x_k - \beta\nu(p^* - p_{k+1})))\right] \end{cases} \tag{6}$$

3 The Deterministic Behavior

In this section we study the model without noise ($\sigma = 0$). Furthermore, without loss of generality, we set $\mu = 0.5$. In this setting we analyze separately the effects of each market component on asset price dynamics. First assume also $\beta = 0$, without fundamental traders in the market. The model (6) turn into the following

$$\begin{cases} p_{k+1} - p_k = \delta x_k \\ x_{k+1} = -\omega x_k + \omega e^{\alpha\delta x_k}(1 - x_k) - \omega e^{-\alpha\delta x_k}(1 + x_k) \end{cases}$$

It is clear that $x = 0$ is an equilibrium for the excess demand dynamic. If there is no excess in demand the asset prices remain in equilibrium. Note that this situation is locally stable only if feedback traders are not enough. Precisely, the stability condition for the excess demand equilibrium $x = 0$ is

$$\alpha\delta < \frac{1}{2}\left(3 + \frac{1}{\omega}\right)$$

This is the only case in which the price process becomes settled around some limit value. It follows that if there are enough positive feedback traders the market is instable. In general it can be proved the existence of a positive number $\alpha^* < \frac{1}{2\delta}(3 + \frac{1}{\omega})$ such that for $\alpha < \alpha^*$ the origin $x = 0$ is the only equilibrium and it is globally stable, for $\alpha = \alpha^*$ there are three equilibria, the origin is locally stable and the other two are symmetrical and instable, for $\alpha^* < \alpha < \frac{1}{2\delta}(3 + \frac{1}{\omega})$ there are two more symmetrical equilibria and limit cycles, finally for $\alpha \ge \frac{1}{2\delta}(3 + \frac{1}{\omega})$ there are three equilibria, the origin becomes instable and the other two are symmetrical and instable.

Now let assume $\alpha = 0$, without feedback traders in the market. In this case (6) turn into

$$\begin{cases} p_{k+1} - p_k = \delta x_k \\ x_{k+1} = -\omega x_k + \omega e^{\beta\nu(p^* - p_{k+1})}(1 - x_k) - \omega e^{-\beta\nu(p^* - p_{k+1})}(1 + x_k) \end{cases}$$

and $(p^*, 0)$ is the only stable equilibrium. Moreover, for generic values of the parameters, this equilibrium is globally stable. This means that the fundamental trading tend to stabilize the market around a fundamental value p^*. The speed of convergence to the equilibrium p^* is increasing on the parameter α. So much quickly in time is the adjustment of prices to the fundamental value as more relevant is the role of fundamental traders in the market.

4 The Global Dynamics

We consider the general setting with all three groups of traders. We have simulated the dynamics of the model (6) for various values of the parameters accordingly to the choice of positives α, β and σ. The purpose of this study is to show the speculative effects of positive feedback traders on the classic market efficiency around the fundamental value. Accordingly to the efficient market assumption, all movements in asset prices can be accounted for news on fundamental values. Viceversa, lots of empirical analysis of time series on real data show that this dynamic rule is not true. Moreover excess returns display positive or negative autocorrelation as a function of the horizon and of the type of trading, see for instance Cutler, Poterba and Summers [4]. The volatility of asset prices

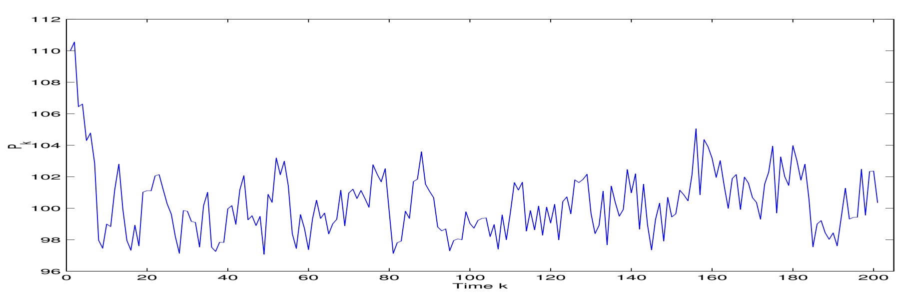

Figure 1: Plot of an asset price trajectory with parameters $\alpha = 2$ and $\beta = 3$.

also is in excess with respect to the classical rational expectation models. Simulations of feedback trading effects in our model confirm that feedback trading may help to explain excess volatility, predictability of stock returns and speculative bubbles growth far away from the fundamental values.

We set $e^{p^*} = 100$, $\omega = 0.2$, $\delta = \mu = \nu = 0.5$ and $\sigma = 0.01$. Initial conditions are $e^{p_0} = 110$ and $x_0 = 0.01$. For each choice of agent parameters α and β our Matlab code generates 1000 trajectories of the processes (p_k) and (x_k) for $0 \le k \le 200$. Figure 1 ($\alpha = 2$ and $\beta = 3$) displays that if fundamental traders have more weight than feedback

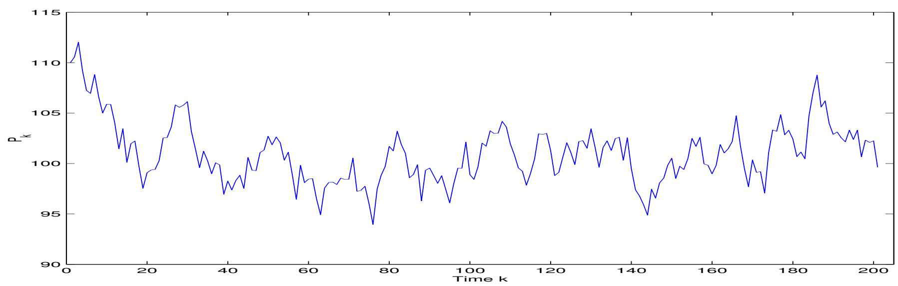

Figure 2: Plot of an asset price trajectory with parameters $\alpha = 3$ and $\beta = 2$.

traders in the market the price will tend to fluctuate around the fundamental value e^{p^*}. The variance of the asset price fluctuation is fuelled up by the positive feedback trading-fundamental traders ratio as showed in Figure 2, where $\alpha = 3$ and $\beta = 2$. Actually, several

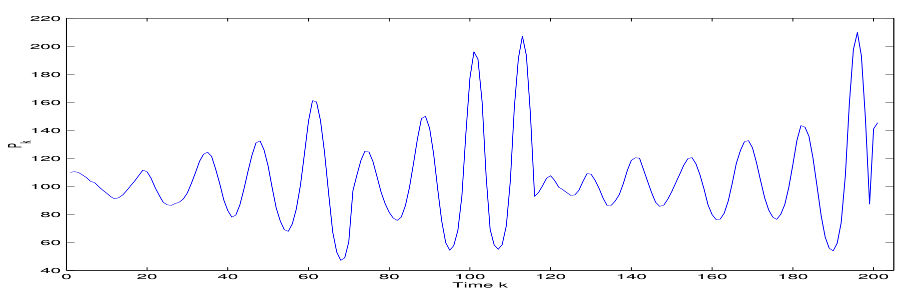

Figure 3: Plot of an asset price trajectory with parameters $\alpha = 8$ and $\beta = 2$.

empirical and theoretical studies agree that negative feedback traders are present during low volatility periods, while positive feedback traders prevail when the volatility is high (see for instance Cutler, Poterba and Summers [4], Sentana and Wahani [12], Balduzzi, Bertola and Foresi [1], and De Long, Shleifer, Summers, and Waldmann [6]). We also observe that in a situation of trading monopoly of feedback traders the asset prices can generate speculative bubbles driving away from the fundamental value, see Figure 3.

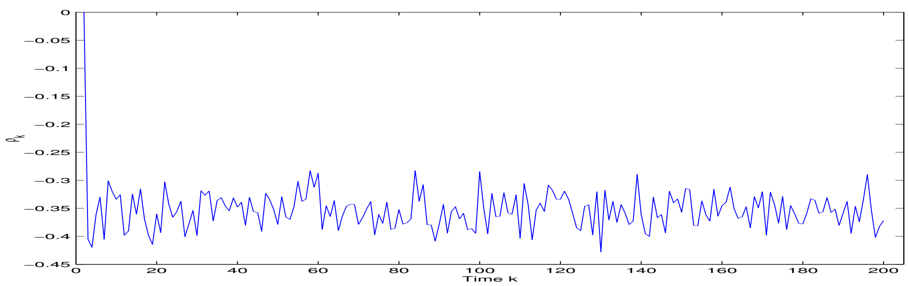

Figure 4: Simulation of the serial correlation in returns with parameters $\alpha = 2$ and $\beta = 3$.

Let now define $r_k = p_k - p_{k-1}$ the logarithmic return of the asset price in the period $(k-1, k)$. We are interested in the analysis of the effects of feedback trading on the excess returns autocorrelation $\rho(k) = corr(r_{k+1}, r_k)$. We recall that in the standard efficient market model the asset prices are log-normal and so $\rho(k) = 0$, no serial correlation of returns is observed. On the other hand the empirical tests shows that the excess returns are somehow predictable and a mean-reverting component, which produces negative autocorrelation over long horizons, or a destabilizing component, which produces positive autocorrelation, are usually observed (see for instance Lo and MacKinlay [9], Campbell, Lo and MacKinlay [3], and Cutler Poterba and Summers [4], Giuli and Monte [8]).

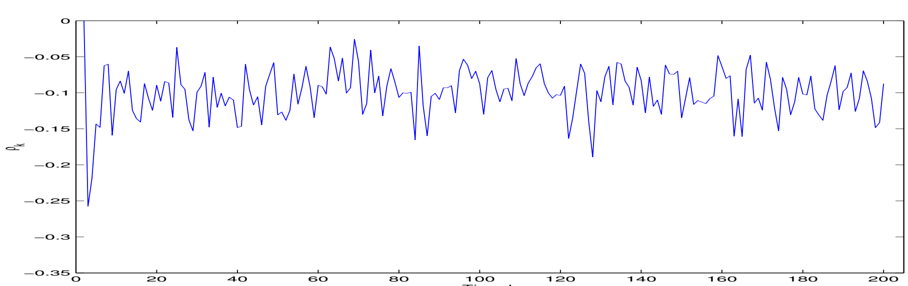

Figure 5: Simulation of the serial correlation in returns with parameters $\alpha = 3$ and $\beta = 2$.

We find again that the behavior of the model depends on the weight of agents in the market. When there are enough fundamental traders in the market the excess returns autocorrelation is negative, Figure 4. The fluctuation level of the excess returns autocorrelation is increasing with the positive feedback trading-fundamental traders ratio, see Figure 5. Finally, when there are not enough fundamental traders, the excess returns autocorrelation is negative over short horizons and positive over long horizons as shows Figure 6.

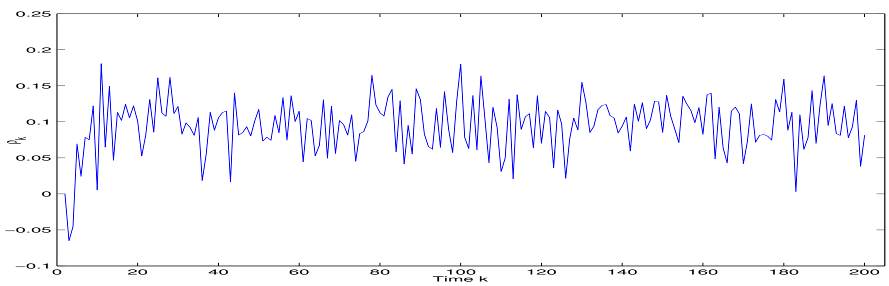

Figure 6: Simulation of the serial correlation in returns with parameters $\alpha = 4$ and $\beta = 2$.

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